In-Network Iterative Distributed Estimation for Power-constrained Wireless Sensor Networks

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Introduction

- Joint sensor selection and routing for distributed estimation
- Estimate-and-forward (EF) approach is used
- Problem is shown to be NP-hard
- A lower bound is presented to compare the performances
Fusion Rule

- **Estimator Update**

\[
\hat{x}[1 : k] = \hat{x}[1 : k-1] + F[k] \left( y_k - h_k^T \hat{x}[1 : k-1] \right)
\]

where

\[
F[k] = \frac{\Sigma_{\hat{x}}[1 : k-1] h_k}{\sigma_k^2 + h_k^T \Sigma_{\hat{x}}[1 : k-1] h_k}
\]

- **MSE Update**

\[
\Sigma_{\hat{x}}[1 : k] = (I - F[k] h_k^T) \Sigma_{\hat{x}}[1 : k-1]
\]
Fusion Rule

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- **MSE Update**

\[
\Sigma_{\hat{x}}[1 : k] = (I - F[k]h_k^T)\Sigma_{\hat{x}}[1 : k-1]
\]
SPT is not the optimal routing structure for distributed estimation when an EF approach is used.
Motivation

- Since $f_c(d_{1,3}) > f_c(d_{1,2})$, total communication cost is reduced.
- Obtained in both the case same quality of estimation.

Distributed

S. Shah, B. B. Lozano
In-Network Iterative Distributed Estimation for Power-constrained WSNs
Problem Formulation

Equivalent form of our convex optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \text{Distortion} \\
\text{subject to} & \quad \text{Comm\_cost} \leq P_{\text{max}} \\
\end{align*}
\]

- \( T \) is a non-spaining tree and a subset of \( G \)
- \( P_{\text{max}} \) is the total power budget allowed
Network Model

- Communication cost model:
  \[ f_c(d_{i,j}) \propto d_{i,j}^\alpha N_0 S_0 B \]

- \( d_{th} \) helps to reduce the complexity of the graph
Signal Model

Linear model is considered:

\[ y_k = h_k x + z_k, \quad k = 1, \ldots, N \]

- \( y \in \mathbb{R} \) is a scalar observation
- \( x \in \mathbb{R} \) is an unknown deterministic scalar parameter to be estimated
- observation of \( x \) is distorted by a scalar \( h_k \in \mathbb{R} \propto 1/d_{k,t}^\beta \)
- and corrupted by additive Gaussian noise \( z_k \in \mathbb{R} \)
- \( z_k \) is assumed to be i.i.d. with pdf \( \mathcal{N}(0, \sigma^2) \)
Parameter Estimation

- Given measurements of the form $y_k = h_k x + z_k$
- Optimal estimator given by the BLUE is:

$$\hat{x} = \frac{\sum_{k=1}^{N} h_k y_k}{\sum_{k=1}^{N} h_k^2}$$

- The associated error covariance matrix is given by:

$$MSE_{\hat{x}} = \left( \frac{\sum_{k=1}^{N} h_k^2}{\sigma^2} \right)^{-1}$$

Minimize $MSE_{\hat{x}}$ subject to a communication cost:

$$MSE_{\hat{x}} = \left( \sum_{k=1}^{N} \frac{b_k h_k^2}{\sigma^2} \right)^{-1}$$

subject to

$$\sum_{k=1}^{N} b_k f_c(d_{k,k_p}) = P_{max} - \Delta$$

$$b_k \leq b_{k_p}, \text{where } k_p = \text{parent of } k$$

$$\Delta \geq 0; b_k \in \{0, 1\}$$

The constraint $b_k \leq b_{k_p}$ ensures that no sensor is selected if its parent on the tree is not selected.
The joint optimization problem of sensor selection and multi-hop routing structure for distributed estimation under a total power constraint, is NP-Hard.

Proof

A polynomial time reduction from the Directed Hamiltonian Path to our problem.
NP-hardness

\[
\text{minimizing } \left( \sum_{k=1}^{n} b_k h_k^2 \right)^{-1} \text{ is equivalent to } - \sum_{k=1}^{n} b_k h_k^2
\]

\[
\min_{\{f_{ij}, b_k\}} - \sum_{k=1}^{n} b_k h_k^2 + \omega \sum_{(i,j) \in E} f_{ij} f_c(d_{i,j})
\]

given \[\sum_{k=p}^{n} b_k h_k^2 = \sum_{(i,j) \in E'} f_{ij} h_j^2 + h_n^2, \text{ above eq. becomes}\]

\[
\min_{\{f_{ij}\}} \sum_{(i,j) \in E'} f_{ij} (\omega f_c(d_{i,j}) - h_j^2) - h_n^2
\]
NP-hardness

- generating a single path from \(i\) to \(S\), i.e. \((i, S)\) DHP is NP-Hard.

- \(DHP \leq_P (i, S)\) DHP

- since \((i, S)\) DHP \(\in\) NP, we can conclude that \((i, S)\) DHP is NP-complete
Joint Sensor Selection and Routing Algorithms

- Fixed-Tree Relaxation-Based Algorithm
- Iterative Distributed Algorithm
Consider the idea of an EF, a relaxed version of the above problem is given by:

\[
\begin{align*}
\text{minimize} & \quad \{b'^r_k, \Delta\} \\
\text{subject to} & \quad \sum_{k=1}^{N} b'^r_k c_k = P_{max} - \Delta \\
& \quad b'^r_k \leq b'^r_{kp} ; \Delta \geq 0 \\
& \quad 0 \leq b'^r_k \leq 1
\end{align*}
\]

where \(b'^r_k\) is the relaxed version of the variable \(b_k\).
Fixed-Tree Relaxation-Based Algorithm

1. Find SPT based only on communication cost
2. Solve relaxed optimization problem to find \( \{ b^r_k \}^N_{k=1} \)
3. Sort the optimal values \( \{ b^r_k \}^N_{k=1} \) in descending order
4. Select the subset of sensors corresponding to the largest \( b^r_k \) while satisfying the power constraint inequality
5. Choose the routing tree \( T \subset SPT \) that span the subset of selected sensors

Then, denoting \( \{ \hat{b}^r_k \}^N_{k=1} \) a binary values such that \( \hat{b}^r_k = 1 \) if \( k \in S \) and \( \hat{b}^r_k = 0 \) if \( k / \in S \), we have that:

\[
L_{fxd} = \left( \sum_{k=1}^{N} \frac{\hat{b}^r_k h^2_k}{\sigma^2} \right)^{-1} \geq p^*
\]
Lower Bound
• set $h_{N+1} = 0$ for $S$ and target location $t$
Lower Bound

\[
\text{minimize } \left\{ b^r_k, \Delta \right\} \\
\text{subject to} \\
\sum_{k=1}^{N+1} b^r_k h_k^2 \sigma^2 \\
\sum_{k=1}^{N+1} b^r_k f_c(d_{k,p}) = P_{\text{max}} - \Delta \\
b^r_{N+1} = 1; \Delta \geq 0 \\
b_k \leq b_{kp} \\
0 \leq b^r_k \leq 1
\]

• take the solution \( \{ b^r_k \}_{k=1}^{n+1} \)
Lower Bound

\[ b_1^r, b_2^r, b_3^r, b_4^r, b_5^r, b_6^r \]

\[
\begin{array}{cccccc}
0.7 & 0.75 & 0.6 & 0.65 & 0.8 & 0.35 \\
\rightarrow & b_7^r & b_8^r & b_9^r & b_{10}^r & b_{N+1}^r \\
\rightarrow & 0.4 & 0.85 & 0.9 & 0.3 & 1
\end{array}
\]

- direction flow between \( i \) and \( j \) by:
  \[ c_{ij} = b_i^r f_c(d_{i,j}) \]
  \[ c_{ji} = b_j^r f_c(d_{i,j}) \]

- if \( c_{ij} < c_{ji} \), then \( j \) will be parent of \( i \), otherwise \( i \) will be parent of \( j \)

- e.g. \( c_{12} < c_{21} \) and \( c_{52} > c_{25} \)
Lower Bound

<table>
<thead>
<tr>
<th>$b_1^r$</th>
<th>$b_2^r$</th>
<th>$b_3^r$</th>
<th>$b_4^r$</th>
<th>$b_5^r$</th>
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</table>

→ $b_7^r$ $b_8^r$ $b_9^r$ $b_{10}^r$ $b_{N+1}^r$
→ 0.4 0.85 0.9 0.3 1

- direction flow between $i$ and $j$ by: $c_{ij} = b_i^r f_c(d_{i,j})$ and $c_{ji} = b_j^r f_c(d_{i,j})$
- if $c_{ij} < c_{ji}$, then $j$ will be parent of $i$, otherwise $i$ will be parent of $j$
- e.g. $c_{12} < c_{21}$ and $c_{52} > c_{25}$
Lower Bound

- Final routing tree routed at $S$
- We update the set $\{k, k_p\}$
- Calculate the lower bound and total communication cost

\[
\text{minimize } \{b^r_k, \Delta\}
\]

$$MSE_\hat{x} = \left( \sum_{k=1}^{N+1} \frac{b^r_k h^2_k}{\sigma^2} \right)^{-1}$$

subject to

$$\sum_{k=1}^{N+1} b^r_k f_c(d_k, k_p) = P_{\text{max}} - \Delta$$

$$b^r_{N+1} = 1; \Delta \geq 0$$

$$b_k \leq b_{kp}$$

$$0 \leq b^r_k \leq 1$$
Iterative Distributed Algorithm

- backbone from $t$ to sink node $S$
- sensor $i$ has $j$ neighbors ($j \in \mathcal{N}(i)$)
- objective function $h_j^{-2} + \gamma_j f_c(d_{i,j})$ at each sensor

- calculate 1-hop neighbors for each $i$
- find best 1-hop neighbors
- update $\gamma_j$: if $h_j > h_{j_{pre}}$ then Decrease $\gamma_{j_{next}}$ else Increase $\gamma_{j_{next}}$
Iterative Distributed Algorithm

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Backbone

$\{a, b, c\} \in T \subset \text{SPT-CC}$

$\{6, 8, 3, 7, 10\} \in \mathcal{N}(c)$
$\{8, 4, 5, 3, 7\} \in \mathcal{N}(b)$
$\{8, 4, 1, 2, 5\} \in \mathcal{N}(a)$
Iterative Distributed Algorithm

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Iterative Distributed Algorithm

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- calculate 1-hop neighbors for each $i$
- find best 1-hop neighbors
- update $\gamma_j$: if $h_j > h_{j_{\text{pre}}}$ then Decrease $\gamma_{j_{\text{next}}}$ else Increase $\gamma_{j_{\text{next}}}$
Backtracking

$p_i$ is the parent sensor of $i$
Backtracking

Compare the link cost and update the parents
Complexity Analysis

Table: Fixed-Tree Relaxation-Based Algorithm

<table>
<thead>
<tr>
<th>SPT-CC</th>
<th>solving relaxed problem</th>
<th>sorting for a subset of sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(MN)$</td>
<td>$O(N^3)$</td>
<td>$O(N \log N)$</td>
</tr>
</tbody>
</table>

total operation is $O(N^3)$

Table: Iterative Distributed Algorithm

<table>
<thead>
<tr>
<th>SPT-CC</th>
<th>outer loop</th>
<th>subset selection operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(MN)$</td>
<td>$O(N)$</td>
<td>$O(KN \log N)$</td>
</tr>
</tbody>
</table>

total operation in this case is $O(N(KN \log N)) = O(KN^2 \log N)$
Results

- 200 sensors deployed randomly in a square region
- Comm. cost model $f_c(d_{i,j}) \propto d_{i,j}^\alpha N_0 S_0 B$ with $\alpha = 4$
- Measurement gain $h_j = 1/d_{j,t}^\beta$ with $\beta = 1$
- Simulated for 100 different network topologies
The subset of activated sensors and associated routing structure routed at the sink node

power budget is given $P_{\text{max}} = 10$
Results

- Comparison with the lower bound $L$
- The bound $L_{fxd}$ is obtained by fixed-tree algo.
- $L_{iter}$ is obtained by iterative distributed algo.
- $L_{ID}$ is obtained from the ID based algorithm
- results are with different $\gamma_j$ values
Results

Gap with the lower bound i.e.
\[ \delta_{ID} = L_{ID} - L, \quad \delta_{fxd} = L_{fxd} - L, \quad \text{and} \]
\[ \delta_{iter} = L_{iter} - L. \]

MSE estimation performance due to scaling for a given fixed power budget \( P_{\text{max}} = 10 \).


References


Thanks

Thank you for your attention!

Questions please!